where we have partitioned T_s into four N by N submatrices. Utilizing the above, it is a matter of straightforward matrix algebra[1] to derive expressions for the elements of T_s:

$$(T_{11})_{ij} = \frac{1}{2\sqrt{1-\lambda^2}\sqrt{r_{i_0} r_{j_l}}} \cdot [\delta_{ij}(r_{i_0} + r_{j_l}) - \lambda(w_{ij} + g_{ij} r_{i_0} r_{j_l})] \quad (11a)$$

$$(T_{22})_{ij} = \frac{1}{2\sqrt{1-\lambda^2}\sqrt{r_{i_0} r_{j_l}}} \cdot [\delta_{ij}(r_{i_0} + r_{j_l}) + \lambda(w_{ij} + g_{ij} r_{i_0} r_{j_l})] \quad (11b)$$

$$(T_{21})_{ij} = \frac{1}{2\sqrt{1-\lambda^2}\sqrt{r_{i_0} r_{j_l}}} \cdot [\delta_{ij}(-r_{i_0} + r_{j_l}) + \lambda(-w_{ij} + g_{ij} r_{i_0} r_{j_l})] \quad (11c)$$

$$(T_{12})_{ij} = \frac{1}{2\sqrt{1-\lambda^2}\sqrt{r_{i_0} r_{j_l}}} \cdot [\delta_{ij}(-r_{i_0} + r_{j_l}) + \lambda(w_{ij} - g_{ij} r_{i_0} r_{j_l})] \quad (11d)$$

where δ_{ij} is the Kronecker delta.

The scattering matrix S relates $\boldsymbol{a}$ to $\boldsymbol{b}$,

$$\boldsymbol{b} = S\boldsymbol{a}. \quad (12)$$

S can be expressed in terms of T_s as follows

$$S = \left[\begin{array}{c|c} T_{12}T_{22}^{-1} & (T_{22}^{-1})' \\ \hline T_{22}^{-1} & -T_{22}^{-1}T_{21} \end{array}\right] \quad (13)$$

where $(T_{22}^{-1})'$ denotes the transpose of T_{22}^{-1} and use has been made of reciprocity.

It is now time to state precisely what we mean by the term "$2N$-port contradirectional coupler." Let us arbitrarily single out conductor 1, and call it the "main line." It is assumed that incident power will be applied only to the main line conductor either at $x=0$ or $x=l$. The other conductors $(2, 3, \cdots, N)$ are called "subsidiary lines." The following requirements are made:

1) When a wave is incident on one of the main line ports, no reflected wave is produced at that port (i.e., the main line ports are matched).
2) When a wave is incident on one of the main line ports, a reflected wave results at the other main line port and at all the subsidiary ports on the same side as the excited port, but no reflected wave results at the subsidiary ports on the opposite side.

The situation is shown schematically in Fig. 3 for the case of excitation at the $x=0$ side. A similar figure applies for excitation at the $x=l$ side. The above requirements translate simply into restrictions on the elements of S. Specifically, we demand an S of the form

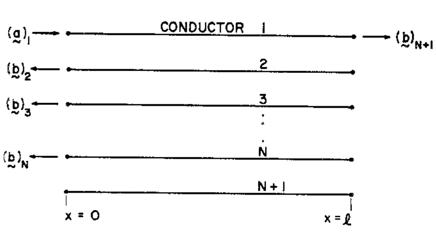

Fig. 3. Multiline directional coupler.

[1] The details are carried out in [4].

$$S = \begin{array}{c} \overbrace{\hspace{6em}}^{N}\ \overbrace{\hspace{6em}}^{N} \\ \left[\begin{array}{cccc|cccc} 0 & X & X\cdots & X & X & 0 & 0\cdots & 0 \\ X & X & X\cdots & X & X & X & X\cdots & X \\ X & X & X\cdots & X & X & X & X\cdots & X \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X & X & X\cdots & X & X & X & X\cdots & X \\ \hline X & X & X\cdots & X & 0 & X & X\cdots & X \\ 0 & X & X\cdots & X & X & X & X\cdots & X \\ 0 & X & X\cdots & X & X & X & X\cdots & X \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & X & X\cdots & X & X & X & X\cdots & X \end{array}\right] \end{array} \begin{array}{l} \left.\vphantom{\begin{array}{c}0\\0\\0\\0\\0\end{array}}\right\} N \\ \left.\vphantom{\begin{array}{c}0\\0\\0\\0\\0\end{array}}\right\} N \end{array}.$$

We note that the inverse of a matrix of the form

$$\begin{bmatrix} X & 0 & 0\cdots & 0 \\ 0 & X & X\cdots & X \\ 0 & X & X & \\ 0 & X & X\cdots & X \\ \vdots & \vdots & \vdots & \vdots \\ 0 & X & X\cdots & X \end{bmatrix}$$

is also of this form. From (13) we see that if T_{22} is in this form, and if $(T_{12})_{11}$ and $(T_{21})_{11}$ are zero, then S will have the desired form. Inspection of (11) shows that all these requirements are met provided that

$$r_{1_0} = r_{1_l} = \sqrt{\frac{w_{11}}{g_{11}}} = \sqrt{\frac{l_{11}}{c_{11}}}, \quad (14)$$

and

$$r_{i_0} = r_{i_l} = -\frac{w_{i1}}{g_{i1}}\sqrt{\frac{g_{11}}{w_{11}}} = -\frac{l_{i1}}{c_{i1}}\sqrt{\frac{c_{11}}{l_{11}}} \quad (15)$$

for $i=2, 3, 4, \cdots, N$. From (5), (6), and (7), all the r's are positive, as required. Since (14) and (15) are independent of λ, the coupler has infinite directivity bandwidth. For $N=2$, (14) and (15) reduce to Oliver's [1] results.

The performance of an experimental 6-port directional coupler is described in [4].

In conclusion, it has been shown that any section of $N+1$ conductor multiline can be made to operate as a $2N$-port directional coupler by the proper choice of the $2N$ load resistances as given in (14) and (15).

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Comments on "Polarization Transformation in Twisted Anisotropic Media"

The artificial anisotropic medium illustrated in Fig. 1 of Van Doeren's Paper[1] is not twistable in the sense intended by the author. A moment's contemplation reveals that the lateral distance moved by the rotating layers, for given distance moved in a direction parallel with the twist axis, is directly proportional to the distance from the twist axis. The result is a medium with characteristics that vary drastically as a function of the position relative to the twist axis.

The inappropriateness of the physical example does not detract from the validity of the theoretical analysis in the body of the paper. The numerical calculations are also valid; they just don't apply to a layered anisotropic medium.

The interesting question that comes out of this is what the artificial twisted anisotropic medium might be, other than a scaled-up replica of a quartz crystal.

Two possibilities are sketched in Fig. 1. The first, shown in Fig. 1(a), involves slicing the layered medium perpendicular to the layers, then rotating each slice by a small angle. The slice thickness and the angle of rotation are the added parameters in this case. The second, shown in Fig. 1(b), involves cutting tubes parallel with the layers, then twisting the layered medium within each tube. The tubes can be circular, hexagonal, or square. If they are not circular, it will be necessary to twist the medium before cutting the tubes. The added parameters are the twist rate of the medium and the tube dimensions.

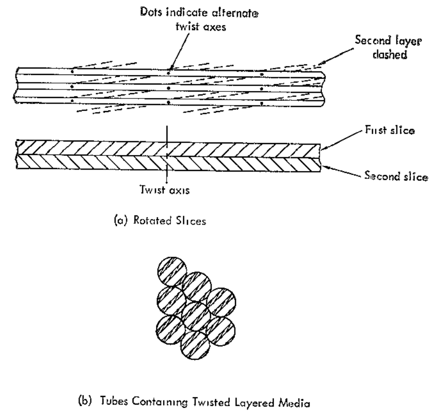

Fig. 1. Possible methods for obtaining artificial twisted anisotropic dielectrics.

There are surely many other possible configurations, but it appears certain that they will also contain periodicities in three dimensions and will therefore, like the two described above, present a challenge to anyone interested in analyzing them.

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